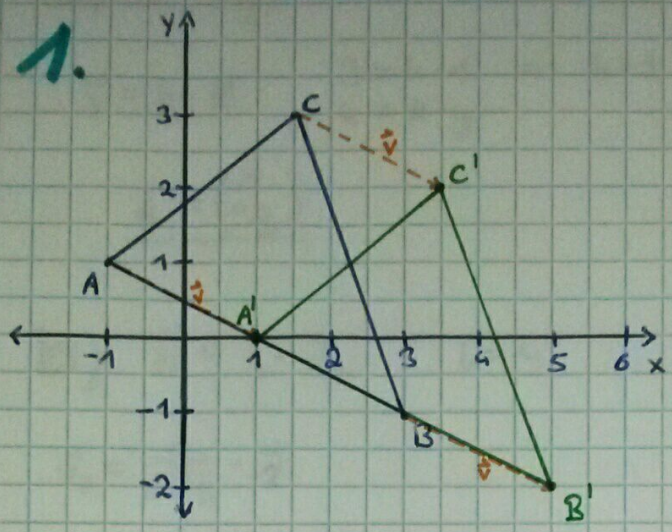
Grundlagen der Vektorrechnung – Beispiele

1.



$A(-1\,|\,1)$

$B(3\,|\,-1)$

$C(1{,}5\,|\,3)$

a) $\vec{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\overrightarrow{OA'} = \overrightarrow{OA} + \vec{v}$ bzw. $A' = A + \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$A'(1\,|\,0)$

$B' = B + \vec{v} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$B'(5\,|\,-2)$

$C' = C + \vec{v} = \begin{pmatrix} 1{,}5 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3{,}5 \\ 2 \end{pmatrix}$

$C'(3{,}5\,|\,2)$

b) $u = \|\overrightarrow{AB}\| + \|\overrightarrow{BC}\| + \|\overrightarrow{AC}\| = \|\overrightarrow{A'B'}\| + \|\overrightarrow{B'C'}\| + \|\overrightarrow{A'C'}\|$

$\overrightarrow{AB} = B - A = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \quad \|\overrightarrow{AB}\| = \sqrt{4^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20}$

$\overrightarrow{BC} = C - B = \begin{pmatrix} 1{,}5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -1{,}5 \\ 4 \end{pmatrix} \quad \|\overrightarrow{BC}\| = \sqrt{\left(\frac{3}{2}\right)^2 + 4^2} = \sqrt{\frac{73}{4}}$

$\overrightarrow{AC} = \begin{pmatrix} 1{,}5 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2{,}5 \\ 2 \end{pmatrix} \quad \|\overrightarrow{AC}\| = \sqrt{\left(\frac{5}{2}\right)^2 + 2^2} = \sqrt{\frac{41}{4}}$

$u = \sqrt{20} + \frac{\sqrt{73}}{2} + \frac{\sqrt{41}}{2} \approx 11{,}95$

2. $\vec{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

a) $\vec{a} = \begin{pmatrix} 1 \\ y \end{pmatrix}$

$\vec{v} = k \cdot \vec{a}$ $\quad \begin{matrix} 3 = k \cdot 1 \\ 4 = k \cdot y \end{matrix} \Rightarrow k = 3$ $\quad \Rightarrow 4 = 3y \rightsquigarrow y = \frac{4}{3}$

$\underline{\vec{a} = \begin{pmatrix} 1 \\ 4/3 \end{pmatrix}}$

b) $\vec{b} = b \cdot \vec{v}_0 \qquad b = 4$

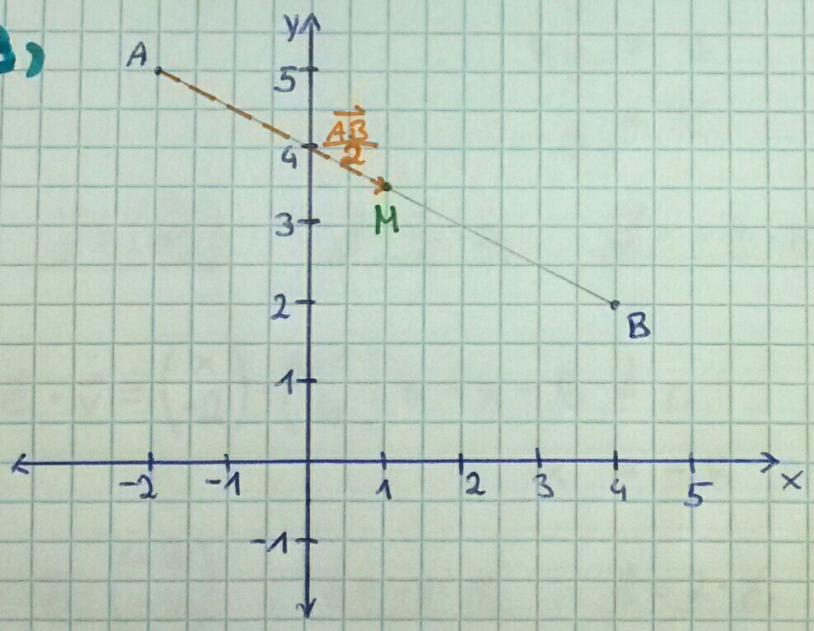
$\vec{v}_0 = \frac{1}{\|\vec{v}\|} \cdot \vec{v}$

$\|\vec{v}\| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

$\vec{v}_0 = \frac{1}{5} \cdot \vec{v} = \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\underline{\vec{b} = \frac{4}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix}}$

3)



A(-2|5)

B(4|2)

$M = A + \frac{\vec{AB}}{2} = B + \frac{\vec{BA}}{2}$

$\vec{AB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix} \qquad \vec{AB} : 2 = \begin{pmatrix} 3 \\ -3/2 \end{pmatrix}$

$M = A + \frac{\vec{AB}}{2} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ -3/2 \end{pmatrix} = \begin{pmatrix} 1 \\ 7/2 \end{pmatrix} \qquad \underline{M(1 / 7/2)}$